Abstract

The concept of q-regular sequences in the sense of Allouche and Shallit is central in various fields of mathematics and computer science. In particular, many sequences from combinatorics and combinatorics of words as well as sequences corresponding to divideand-conquer approaches fit in the framework of q-regular sequences. This framework has been studied by various authors over the last three decades and provides a useful theory for the study of those sequences.

In this thesis we study a special class of q-regular sequences, the so-called q-recursive sequences, and the asymptotic behaviour of their summatory functions. q-Recursive sequences are sequences which satisfy a specific type of recurrence relation: A subsequence of indices modulo q^M equals a linear combination of subsequences of indices modulo q^m for some m < M. It turns out that this property is quite natural and many combinatorial sequences are in fact q-recursive.

After a general study of this type of sequences and the proof that q-recursive sequences are indeed q-regular, we derive asymptotic results for q-recursive sequences by using a result from Heuberger and Krenn on the asymptotic analysis of regular sequences. Finally, we complete this work by illustrating the concepts and our results in three specific examples of q-recursive sequences: We analyse Stern's diatomic sequence, the number of non-zero elements in some generalized Pascal's triangle and the number of unbordered factors in the Thue–Morse sequence.